**Mathematics** **Depth-of-Knowledge-Levels**

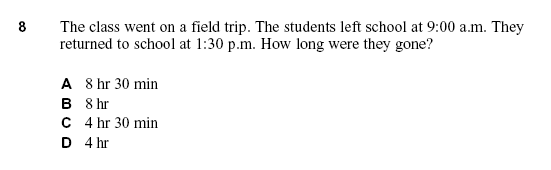
**Examples**

Sample Mathematics Assessment Items

Now try coding some sample assessment items using the Mathematics DOK Levels. The following are sample items for three different grade levels. After you are finished coding these, read our “Answers” on the page that follows.

**[The following five items are from Grade 4 mathematics assessments]**

**1)**



**2)** Sam, Terry, and Kim each own some baseball cards that Ted is willing to trade them for. Here is what they are worth:

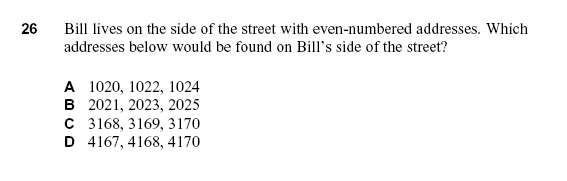
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Sam’s cards: | |  | Tetsuo’s cards: | |  | Kim’s cards: | |
|  |  |  |  |  |  |  |  |
| Bret Boone | $0.80 |  | Sammy Sosa | $1.30 |  | Randy Johnson | $0.70 |
| Andres Galarraga | $0.40 |  | Greg Maddux | $1.00 |  | Barry Bonds | $2.30 |
| Mark McGuire | $1.50 |  |  |  |  |  |  |

Ted will trade his Alex Rodriguez card for $6.75 worth of cards. What is the best trade that Sam, Tetsuo, and Kim can make for Ted’s Alex Rodriguez card?

What trade could Sam, Tetsuo, and Kim offer Ted that would be the most fair between Sam, Tetsuo, and Kim?

Explain your thinking and show all your work.

**3)**



**4)**

|  |  |
| --- | --- |
| Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work. | |
|  | Laura wanted to enter the number 8375 into her calculator. By mistake, she entered the number 8275. Without clearing the calculator, how could she correct her mistake? |

Without clearing the calculator, how could she correct her mistake another way?

**5)**

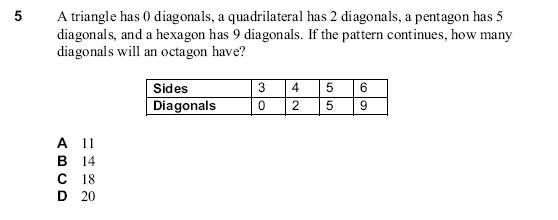
|  |  |  |
| --- | --- | --- |
| Stimulus Image | | |
|  | Based on the map above, about how many miles is the shortest route from Oakdale to Fenton? | |
|  |  |  |
|  | A) | 100 |
|  | B) | 70 |
|  | C) | 40 |
|  | D) | 20 |

**[The following five items are from Grade 8 assessments]**

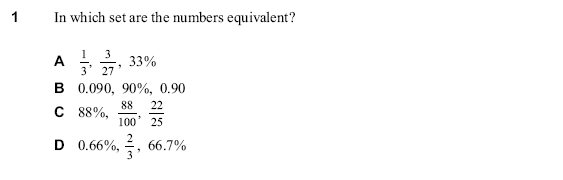
**6)**

|  |  |
| --- | --- |
| From any vertex of a 4-sided polygon, 1 diagonal can be drawn. From any vertex of a 5-sided polygon, 2 diagonals can be drawn. From any vertex of a 6-sided polygon, 3 diagonals can be drawn. From any vertex of a 7-sided polygon, 4 diagonals can be drawn. | |
| How many diagonals can be drawn from any vertex of a 20-sided polygon? |

**7)**

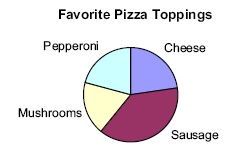


**8)**



**9)**

The school newspaper conducted a survey about which ingredient was most preferred as a pizza topping. This graph appeared in the newspaper article.



What information would best help you determine the number of people surveyed who preferred sausage?

**A** number of people surveyed and type of survey used

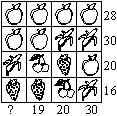
**B** type of survey used and ages of people surveyed

**C** percent values shown on chart and number of people surveyed

**D** ages of people surveyed and percent values shown on chart

**10)**

Look at the drawing. The numbers alongside each column and row are the total of the values of the symbols within each column and row. What should replace the question mark?



1. 23
2. 25
3. 28
4. 30
5. 32

**[The following five items are from Grade 11 assessments]**

**11)** Which of the following is NOT true for any value of *x*?

**A** *x < x² < x³*

**B** *x³ < x < x²*

**C** *x² < x < x³*

**D** *x < x³ < x²*

**E** x³ < x² < x

**12)**

Players A and B are playing a game. On a table between them is a stack of *n* pennies. First, Player A removes either one or two pennies from the stack. Then Player B removes either one or two pennies from the stack. They alternate in this way until no pennies remain. The loser is the player who removes the last penny from the stack.

If they start the game with 5 pennies in the stack, how many pennies should Player A take from the stack on her first turn? Why?

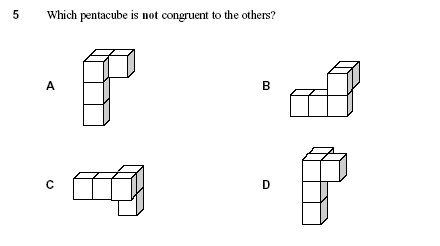
If the game starts with 7 pennies in the stack, would you rather be Player A or B? Why?

For what values of *n*, if any, is it best to be player A?

For what values of *n*, if any, is it best to be player B?

Explain and justify your answers.

**13)**



**14)**

|  |  |
| --- | --- |
|  | One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with income greater than $10,000 to pay a tax of 6 percent only on the part of their income that exceeds $10,000.  A person's effective tax rate is defined as the percent of total income that is paid in tax.  Based on this definition, could any person's effective tax rate be 5 percent? Could it be 6 percent? Explain your answer. Include examples if necessary to justify your conclusions. |

**15)**

*S = a/b + c/d*

If 0 < *a*  < *b*< *c* < *d*  in the equation above, then the greatest increase in *S* would result from adding 1 to the value of which variable?

(A) *a*  
(B) *b*  
(C) *c*  
(D) *d*  
(E) *There is not enough information to know for certain.*

**DOK Levels for the Sample Mathematics Assessment Items**

**Grade 4 Items:**

1) Level 2. The choices offered indicate that this item is intended to identify students who would simply subtract 9 minus 1 to get an 8. More than one step is required here. The students must first recognize the difference between a.m. and p.m. and make some decisions about how to make this into a subtraction problem, then do the subtraction.

2) Level 4. This is a complex open-ended problem requiring students “to make several connections and apply one approach among many.” It requires the students to plan and organize, and to weigh solutions based on different kinds of criteria. Students should be allowed at least 20 minutes for this problem, which is an extended period of time for a test item.

3) Level 1. Students only need to be able to recognize even numbers.

4) Level 3.  “An activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.” Since there are multiple possible approaches to this problem, the student must make strategic decisions about how to proceed, which is more cognitively complex than simply applying a set procedure or skill.

5) Level 1. This measurement item requires no analysis of the map itself, since the route in question is a straight line. If the line were not straight, this item would require estimation and perhaps even calculation, making it Level 2 or 3.

**Grade 8 Items:**

6) Level 1. The first thing to note is that this is not really a geometry item. Rather, it simply requires students to notice an easy, routine pattern. DOK levels are difficult to assign for many pattern-recognition problems, because they depend on how routine the pattern is. This particular pattern is immediately recognizable and requires no processing, but a more complex pattern could make this a Level 2 or even Level 3 item.

7) Level 2. This item is included in order to contrast it with the previous item. Pattern recognition is required, but the non-routine nature of this pattern brings the item up to a higher DOK level. Some analysis and generalization is required in order to understand and extend this pattern.

8) Level 2. There are a number of different concepts and procedures that can be used for this problem, rather than an obvious, simple one. Students must not only be able to *identify* different representations of rational numbers (Level 1), but also to *manipulate* and *compare* these representations (Level 2). This means that numerous interdependent and non-trivial steps are involved here. However, this does not require any conjecturing, planning, abstracting, or explaining, so it is not Level 3.

This item demonstrates the importance of knowing whether calculators are allowed on the examinations or not. If a calculator were allowed on this examination, this would clearly be Level 1, instead of Level 2.

9) Level 2. This is an example of how a problem that is multiple choice can reduce its DOK level. If the multiple choices were removed here and the problem were left open-ended, it would be Level 3. But here the student need only weigh the options against one another, easily discarding “type of survey used” and “ages of people surveyed” as bogeys. So they can easily determine that C is immediately better than A or D, without even having to think analytically or creatively about why percent values shown or number of people surveyed would be important information to know.

10) Level 3. This item can be approached through a number of viable strategies: pattern recognition, guess-and-check, algebra, etc. This freedom means that the student must make choices and assumptions. Furthermore, no matter what strategy she employs, she must keep track of a complex logical chain. The multiple choices provided do not make this task any less complex.

**Grade 11 Items:**

11) Level 3. This is another example of an item that is at Level 3 without being open-ended. This item requires generalization, reasoning, and hypothesis testing, involving some creativity in choosing examples that test the hypotheses.

12) Level 4. This problem requires students to form game strategies, create data, notice number patterns, and justify how and why those patterns arise. It involves inductive, deductive, and strategic reasoning over an extended period of time, perhaps 30 minutes. This may even be a problem best done in pairs or groups within a testing environment.

13) Level 2. This item is not Level 1 because it is not routine, nor does it focus on a memorized definition or procedure. In fact, it involves numerous steps, because it requires students to compare several different pairs of shapes before arriving at the correct answer. For these reasons, many spatial reasoning items are Level 2.

Note that this may be a coded as a source-of-challenge item, because choice C seems to be drawn in a misleading way.

14) Level 3. This item gives the student a new definition and asks her to use it as a basis for reasoning. In order to ascertain whether the student really understands the asymptotic behavior that makes a 6% effective rate impossible, this item *must* be open-ended. This is why most Level 3 items are open-response items, because the complexity of thinking they require the students to display could not be displayed using the multiple choice option.

15) Level 3. If a multiple-choice item is Level 3, often it is because the multiple choices do not constrain or guide the possible solutions. The choices here allow for *all* possible responses to this item, including the response that the problem cannot be solved. This gives such an item the character of an open-ended item, even though it is not one